

prepared by K. O. May. A recent translation of the complete extant correspondence between Euler and the Göttingen astronomer T. Mayer appears in Eric G. Forbes, *The Euler-Mayer correspondence* (1751-1755), 1971, (Macmillan).

2. "vademecum (L. 'go with me') 1. A book or manual suitable for carrying about with one for ready reference. 2. A thing commonly carried about by a person as being of some service to him." *Oxford Universal Dictionary* (3rd ed., 1944, repr. 1955).

3. Gustaf Eneström, *Verzeichnis der Schriften Leonhard Eulers*. Erste Lieferung, Leipzig, 1910; zweite Lieferung, Leipzig, 1913 = *Ergänzungsband IV, Jahresbericht der Deutschen Mathematiker-Vereinigung*. This Stockholm mathematician tabulated Euler's works according to date of publication, date of composition and subject matter.

MESSAGE D'UN MATHÉMATICIEN: HENRI LEBESGUE. By Lucienne Félix. Paris (Albert Blanchard). 1974. 259 p. 65f.

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Henri Lebesgue (1875-1941) published in 1902 his doctoral thesis, *Intégrale, Longueur, Aire*, surely one of the best theses ever written. Now, almost three-quarters of a century later, the Lebesgue integral is recognized as one of the most important mathematical developments of this century.

The major part of this book consists of selections, mostly nontechnical, from Lebesgue's writings. These are edited by Mademoiselle Lucienne Félix, who has written connecting passages and also several introductory chapters. She was a student of Lebesgue at the *École Normale Supérieure* at Sèvres, then his assistant there for eight years, and finally editor of his last book, *On Geometric Constructions*.

Chapter I is divided into three sections, Biography, The Man, and The Professor, which contain many personal reminiscences of Mademoiselle Félix. The reader is given a vivid portrayal of a modest and unassuming man, somewhat shy and awkward, compassionate, and of great integrity. One will long remember her picture of the dying man, in the occupied Paris of 1941. During the period that Lebesgue taught at Sèvres (1921-1938), he was also a professor at the *Collège de France*. He had held previous positions at the universities in Rennes, Poitiers, and Paris. Almost all the anecdotes in "The Professor," however, are concerned with his teaching at Sèvres. (The *normalienne* at Sèvres was indeed fortunate to have had such eminent mathematicians as Lebesgue, Picard and Borel as professors!)

Chapter II (The Philosophy of the Mathematician) consists of excerpts from Lebesgue's writings and of reports of a

conference on the foundations of mathematics held in Zurich in 1938. Particularly interesting were Lebesgue's repeatedly expressed misgivings about the validity of nonconstructive proofs and, especially, of proofs which depend on the Axiom of Choice.

Chapter III (Research and Discovery) begins with two sections on research in general and on Lebesgue's explanations of his motivations in research problems. His lifelong interest in pure geometry was important. He wrote, "Toutes mes recherches ont ce caractère commun de procéder d'une vue directe et, en quelque sort, géométrique des problèmes étudiés..."

Next are sections on particular topics in Lebesgue's research, beginning with nine pages on real analysis, measure theory, and integration. First are general comments. He stressed the fact that in his integral it is subdivisions of the range of the function, rather than of the domain, which are used in forming the approximating sums. Next is a section on the unfortunate dispute with Borel over priority.

This reviewer finds these sections somewhat disappointing and would have preferred a longer discussion of Lebesgue's most important discovery. More might have been said about the defects of the Riemann integral and of earlier efforts to remedy these defects, about the motivations of Lebesgue in his discoveries, about the more important theorems on integration due to Lebesgue, and about the significance of the Lebesgue integral for later mathematics. For this sort of discussion the historical notes of Bourbaki or the book by T. H. Hawkins should be consulted. Also in this chapter are sections about Lebesgue's work on the Weierstrass Theorem (concerning uniform approximation of a continuous function by polynomials), applicable surfaces, dimension, polyhedra, and a theorem on the geometry of the triangle due to Morley.

Chapter IV (Lebesgue, Historian of Mathematics) begins with a collection of general remarks on history. These are followed by sections on the history of trigonometric series and of imaginaries. Next is an interesting section in which Lebesgue strongly protests statements by Klein, which characterized the work of Hermite and Lindemann (on the transcendence of  $e$  and  $\pi$ ) as "very complicated" and spoke of the great simplifications made by Weierstrass, Hilbert, Hurwitz, and Gordon. Lebesgue felt that this characterization was incorrect and implied that German nationalism may have influenced Klein's opinions. This chapter ends with a section on individual mathematicians--Vandermonde, whose work Lebesgue felt was greatly underrated; Jordan, whom Lebesgue knew and admired; and finally Roberval and Ramus, two early predecessors of Lebesgue at the *Collège de France*.

The last three chapters (Didactic Works, Texts on Education of Teachers, and Courses at the *École Normale Supérieure* at Sèvres) are probably of less interest to the general reader than the rest of the book. However, it is evident that Lebesgue

thought deeply about mathematical education, and these chapters can give new insights into a number of different topics and better ways to present them to a class.

Mademoiselle Félix has rendered a great service to the mathematical community in writing and editing this fine book.

STABILITY OF MOTION. Edited by A. T. Fuller. London (Taylor and Francis), New York and Toronto (John Wiley). 1975.

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E. J. Routh had been a Tripos coach under the old Cambridge system for twenty years when the Adams Prize subject *The criterion of dynamical stability* was proposed. The hard grind had not blunted his originality, but perhaps encouraged the thoroughness of his winning essay. It is valuable to have it reprinted here in full (120 pages) for it is very hard to come by; the corresponding sixth chapter in volume 2 of his rigid dynamics is also set out at length (and is not much easier to find in the original). Routh's criterion for stability is in terms of the linear approximation to the disturbed motion, and so the major part of his essay lies in the methods for determining the nature of the roots of an algebraic equation. One chapter does discuss briefly situations in which the linear approximation may prove inadequate (the case of 'small denominators', or resonance between second-order and first-order terms). Oddly enough Routh pays no attention to the case where the first-order terms happen to vanish identically; and of course it would be quite out of keeping with the Cambridge analytical tradition of the nineteenth century to expect any look forward to more general stability ideas like those of Lyapounov.

Routh had already considered the criteria, in terms of the roots of a biquadratic, which determine the stability of (linear) small oscillations of a system with two degrees of freedom and his papers on this and on the stability of the Laplace three-particle configuration, together with an earlier verbal comment by W. K. Clifford (suggesting, as a method of determining when all the real parts of the roots of an equation are negative, forming the equation whose roots are the sums of the roots taken in pairs and finding when its real roots are all negative), are reprinted from the London Mathematical Society's Proceedings (1874, 75 & 68). Clifford's suggestion was followed up by Routh in the essay. The book also contains a 1836 paper of Sturm's (from Liouville), which the editor believes to have been known to Routh, and which proves the generalization of the Sturm division process for complex equations--a generalization already proved by Cauchy in 1831, since it is nothing but his